How simulating exposure time variations in the LIP Model. 
Application: moving objects acquisition.

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Introduction
The LIP (Logarithmic Image Processing) Model offers a framework consistent with Human Vision. Based on the transmittance law, the LIP operators apply to images acquired in transmission or reflection. In this paper, we extend to colour images a result previously established for grey level images which simulated variable exposure times by means of a LIP subtraction. The quality of such a stabilization is evaluated thanks to the Euclidean metric. An application is then proposed: it consists in the fast acquisition of a moving object resulting in a dark image without blur. By simulating a larger acquisition time, we produce an image both enhanced and de-blurred to be compared with the initial one ...

I- Introduction, notations and recalls
The LIP (Logarithmic Image Processing) Model has been introduced by Jourlin et al ([1], [2], [3]). It proposed first a physical and mathematical framework for processing grey level images acquired in transmission (when the observed object is placed between the source and the sensor). Based on the transmittance law, two operators on images were defined, allowing the addition of two images and the multiplication of an image by a real number, each of them resulting in a novel grey level image. Such operators possess strong mathematical properties, as recalled hereafter. Furthermore, the demonstration by Brailean ([4]), of the LIP Model consistency with human vision, considerably enlarges the application field of the Model, particularly for images acquired in reflected light on which we aim at simulating human visual interpretation. The Model has been extended to colour images, under the name of LIPC ([5]).

A grey level image f is defined on a spatial support D, with values in the grey scale [0, M[, which may be written :

\[ f: D \subset \mathbb{R}^2 \rightarrow [0, M] \subset \mathbb{R} \]

Remark 1: in the LIP Model, 0 corresponds to the « white » extremity of the grey scale, which means to the source intensity, i.e. when no obstacle (object) is placed between the source and the sensor. The reason of this grey scale inversion is that 0 will appear as the neutral element of the addition law defined in formula (1). The other extremity M is a limit situation where no element of the source is transmitted (black value). Thus this value is excluded from the scale, which means that for 8-bits digitized images, the 256 grey levels correspond to the interval of integers [0:255].

The addition of two images f and g corresponds to the superposition of the obstacles (objects) generating respectively f and g and is noted \( f \oplus g \). Its expression is as follows:

\[ f \oplus g = f + g - \frac{f \cdot g}{M} \quad (1) \]
From this addition law, it is possible ([1], [3]) to derive the multiplication of an image by a real number \( \lambda \) according to:

\[
\lambda \triangle f = M - M(1 - \frac{f}{M})^\lambda \quad (2)
\]

Remark 2: if \( f(x, y) \geq g(x, y) \) \( \forall (x, y) \in D \), it is possible to define the subtraction \( f \triangle g \) according to:

\[
f \triangle g = \frac{f - g}{1 - \frac{g}{M}} \quad (3)
\]

Applied in the particular case where \( g \) is a constant, this subtraction operator will appear in the next section as the central logarithmic tool to perform simulations of exposure time variations.

For more information concerning the LIP Model, the interested reader can refer to [6] for example.

II- Simulation of exposure time for grey level and colour images

1- For grey level images

In a previous paper, Carré et al. ([6]) demonstrated the ability of the LIP subtraction of a grey level image \( f \) by a constant \( C \) to simulate exposure time changing (Fig. 1).

![Figure 1. LIP subtraction simulating exposure time variation: a) f: image acquired at 10 ms, b) g: image acquired at 100 ms, c) f \triangle C, where C is computed in order to simulate an image acquired at 100 ms.](image)

Remark: in the previous figure (c), \( C \) is choosen such that the average grey level value of \( f \triangle C \) and \( g \) are equal.

2- Extension to colour images

Now let us consider a colour chart acquired under variable exposure times (Fig. 2).

![Figure 2. Images of a colour chart with variable exposure times: 3ms, 6ms, 18ms, 24ms, 33ms and 45ms from left to right.](image)

Now, let us choose the image acquired at 33ms as a target. We wish to transform each image of Fig. 2 by simulating an exposure time of 33ms.

To reach this goal, we will first associate to each image \( f \) of Fig. 2 an enhanced one, noted \( f \triangle C(f) \). If \( h \) designs the histogram and \( \sigma \) the standard deviation, \( C(f) \) is computed as follows:

\[
\sigma[h(f \triangle C(f))] = \sup_{C \in [0, M]} \sigma[h(f \triangle C)]
\]

Contrary to a method of histogram equalization for example, this approach permits to get an enhanced image independent of the initial exposure time (Fig. 3).
Let \( \{I_i\}_{i \in J} \) design a family of images of a same scene taken under different exposure times \( t_i \).

For each image \( I_i \), we can find the constant \( C_i \) defined by the formula:

\[
\sigma[h(I_i \triangle C_i(t_i))] = \sup_{C \in [0, M]} \sigma[h(I_i \triangle C)].
\]

If \( H_i = I_i \triangle C_i \) denotes the enhanced image of \( I_i \), we can show that \( \forall i, \forall j, H_i = H_j = H \). For a pair \( (i, j) \in J^2 \), we can write \( H = I_i \triangle C_i = I_j \triangle C_j \), yielding:

\[
I_j = I_i \triangle (C_i \triangle C_j) \quad \text{equation (I)}.
\]

We noticed that the function \( C=f(t) \) -where \( C \) is the constant maximizing the variance of the image and \( t \) its exposure time- is a linear function as shown in Figure 4. Finally, it is possible to deduct an exposure time from another by using the linear relation and the equation (I).

### III- Discussion

The visual quality of the previous simulations is very satisfying. Nevertheless, for very-low light images (3ms for example), the enhancement applies to a noisy image. Thus it seemed us interesting to perform an objective evaluation of the “distance” between the result and the target. We selected the following 4 colours \( C_1, C_2, C_3 \) and \( C_4 \) (Fig. 6) in the target image (33ms). Their Euclidean distances with the corresponding colours in the simulations (at 33ms) of images acquired at 3, 6, 18 and 45ms have been computed, the calculus being done on the average colours inside each path in order to avoid problems due to the noise. The following Table 1 groups the results where the Red, Green and Blue components of each colour are listed. The last column (Euclidean distances) shows a good quality of the exposure time simulation. Note that the distance increases for very low-light images (3ms) and that the dominant components of simulated colours are well estimated.

![Figure 6. Four colours issued from the enhanced image of the chart](image-url)
### Table 1. RGB components (Target and images simulated at 33ms). Last column: Euclidean distances

<table>
<thead>
<tr>
<th>Time</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>Euclidean distances</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 to 33ms</td>
<td>[19.3;12.7;57.5]</td>
<td>[86.3;36.9;57.0]</td>
<td>[125.2;76.9;9.54]</td>
<td>[90.3;35.4;12.0]</td>
<td>13.14</td>
</tr>
<tr>
<td>6 to 33ms</td>
<td>[20.9;17.4;56.1]</td>
<td>[89.9;43.6;59.2]</td>
<td>[132.0;86.3;6.61]</td>
<td>[94.7;42.9;7.5]</td>
<td>6.00</td>
</tr>
<tr>
<td>18 to 33ms</td>
<td>[17.9;19.3;55.0]</td>
<td>[88.0;45.2;59.6]</td>
<td>[132.3;91.3;1.07]</td>
<td>[94.8;45.7;1.2]</td>
<td>1.05</td>
</tr>
<tr>
<td>24 to 33ms</td>
<td>[17.6;19.3;54.3]</td>
<td>[87.4;45.2;59.0]</td>
<td>[131.6;91.8;1.01]</td>
<td>[93.9;45.8;1.2]</td>
<td>0.96</td>
</tr>
<tr>
<td>33ms (Target)</td>
<td>[17.6;19.9;54.9]</td>
<td>[87.5;45.9;59.4]</td>
<td>[131.4;91.8;0.04]</td>
<td>[94.3;46.5;0.6]</td>
<td>0.00</td>
</tr>
<tr>
<td>45 to 33ms</td>
<td>[17.4;20.0;55.4]</td>
<td>[89.7;47.0;60.6]</td>
<td>[133.7;93.2;0.01]</td>
<td>[96.1;47.6;0.1]</td>
<td>2.06</td>
</tr>
</tbody>
</table>

### IV- Application: acquisition of moving objects

In this section, we present an example of moving object. A reprint of the Congress label (Fig. 7-a) is submitted to a rotation (3.75 revolutions per second) (Fig. 7-b). Acquired at 30ms, the image appears strongly blurred (7-b). A reduction of the exposure time (2ms) results in a very low-light image (7-c), on which we simulate an exposure time of 30ms producing a “readable” image (7-d).

Figure 7. Acquisition of a rotating object.

### Conclusion and perspectives

In the present paper, we proved the ability of Logarithmic subtraction to simulate variable exposure times for grey level and colour images. The same tool also permits to perform image stabilization in presence of variable lighting. For moving objects, a short exposure time must be selected resulting in a dark image which is enhanced by simulation of a longer exposure time. In a future work, we will apply local processing to get at the same time an enhancement of the dark regions and a darkening of the bright ones. It would be also interesting to test our technique in situation of High Dynamic Range images.

### References